MATHEMATICS-2015
Class-XII

Section – A

Q. 1. Write the value of $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

Sol.

$$\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

$[\textbf{Applying } R_1 \rightarrow R_1 + R_2$

Taking common $(x+y+z)$ and -3 from R_1 and R_3 respectively

$$= -3(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$(\because \textbf{R}_1 \textbf{and R}_3 \textbf{ are identical})$

Q. 2. Write sum of the order and degree of the following differential equation :

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$$

Sol.

Order $= 2$ Degree $= 1$

$\therefore$ Sum $= 2 + 1 = \mathbf{3}$

Q. 3. Write the integrating factor of the following differential equation :

$$(1+y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

Sol.

$$(1+y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

$$(2xy - \cot y)\frac{dy}{dx} = -(1+y^2)$$

$$2xy - \cot y = -(1+y^2)\frac{dx}{dy}$$

$$\frac{2xy}{-(1+y^2)} + \frac{\cot y}{1+y^2} = \frac{dx}{dy}$$

$$\frac{dx}{dy} + \frac{2xy}{1+y^2} = \frac{\cot y}{1+y^2}$$

Comparing with $\frac{dx}{dy} + \mathrm{P}x = \mathrm{Q}$

$$\mathrm{P} = \frac{2y}{1+y^2}, \mathrm{Q} = \frac{-\cot y}{1+y^2}$$

Integrating factor $(\mathrm{I.F.}) = e^{\int \mathrm{P}\, dy}$

$= e^{\int \frac{2y}{1+y^2}dy}$

$= e^{\int \frac{dA}{A}}$ [Let $A = 1 + y^2, dA = 2y\,dy$

$= e^{\int \log(A)}$

$= \mathbf{A} = (\mathbf{1} + \boldsymbol{y}^{\mathbf{2}})$

Q. 4. If $\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{c}}$ are mutually perpendicular unit vectors, then find the value of $|\mathbf{2}\hat{\boldsymbol{a}} + \hat{\boldsymbol{b}} + \hat{\boldsymbol{c}}|$.

Sol.

$\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors,

$\hat{a}\,.\,\hat{b} = 0, \quad |\hat{a}| = |\hat{b}| = |\hat{c}| = 1 \quad \ldots (i)$

$\hat{b}\,.\,\hat{c} = 0, \quad \hat{c}\,.\,\hat{a} = 0$

$|2\hat{a} + \hat{b} + \hat{c}|^2 = (2\hat{a} + \hat{b} + \hat{c})^2$

$= (2\hat{a} + \hat{b} + \hat{c}).(2\hat{a} + \hat{b} + \hat{c})$

$= 4\hat{a}^2 + 2\hat{a}\hat{b} + 2\hat{a}\hat{c} + 2\hat{b}\hat{a} + \hat{b}^2 + \hat{b}\hat{c} + 2\hat{c}\hat{a} + \hat{c}\hat{b} + \hat{c}^2$

$= 4(1) + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 1$ From (i)

and $\begin{bmatrix} \hat{a}^2 = |\hat{a}|^2 = 1^2 = 1 \\ \hat{b}^2 = |\hat{b}|^2 = 1 \\ \hat{c}^2 = |\hat{c}|^2 = 1 \end{bmatrix}$

$= 6|2\hat{a} + \hat{b} + \hat{c}| = \sqrt{\mathbf{6}}$ **(squaring root on both sides)**

Q. 5. Write a unit vector perpendicular to both the vectors $\vec{\boldsymbol{a}} = \hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = \hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$.

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$= \hat{\imath}(-1) - \hat{\jmath}(-1) + \hat{k}(0)$

$= -\hat{\imath} + \hat{\jmath}$

$\vec{a} \times \vec{b} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

A unit vector $\perp$ to both $\vec{a}$ and $\vec{b} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$= \frac{-\hat{\imath}+\hat{\jmath}}{\sqrt{2}} = \frac{-\mathbf{1}}{\sqrt{\mathbf{2}}}\hat{\boldsymbol{\imath}} + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\hat{\boldsymbol{\jmath}}$

Q. 6. The equation of a line are $\mathbf{5}\boldsymbol{x} - \mathbf{3} = \mathbf{15}\boldsymbol{y} + \mathbf{7} = \mathbf{3} - \mathbf{10}\boldsymbol{z}$. Write the direction cosines of the line.

Sol.

$5x - 3 = 15y + 7 = 3 - 10z$

$5\left(x - \frac{3}{5}\right) = 15\left(y + \frac{7}{15}\right) = -10\left(z - \frac{3}{10}\right)$ [LCM of 5,10,15 is 30

Dividing throughout by 30, we have

$$\frac{\cancel{5}\left(x-\frac{3}{5}\right)}{\cancel{30}_6}=\frac{\cancel{15}\left(y+\frac{7}{15}\right)}{\cancel{30}_2}=\frac{\cancel{-10}\left(z-\frac{3}{10}\right)}{\cancel{30}_3}$$

Direction ratios of given line are $6, 2, -3$

$a=6, b=2, c=-3$

$\sqrt{a^2+b^2+c^2}=\sqrt{6^2+2^2+(-3)^2}=7$

Direction cosines are

$l=\frac{a}{\sqrt{a^2+b^2+c^2}}$	$m=\frac{b}{\sqrt{a^2+b^2+c^2}}$	$n=\frac{c}{\sqrt{a^2+b^2+c^2}}$
$=\mathbf{\frac{6}{7}}$	$=\mathbf{\frac{2}{7}}$	$=\mathbf{\frac{-3}{7}}$

Section – B

Q. 7. To promote the making of toilets for women, an organisation tried to generate awareness though (i) House call, (ii) letters and (iii) announcements. The cost of each mode per attempt is given below :

(i) Rs. 50 (ii) Rs. 20 (iii) Rs. 40

The number of attempts made in tree villages X, Y and Z are given below :

	(i)	(ii)	(iii)
X	**400**	**300**	**100**
Y	**300**	**250**	**75**
Z	**500**	**400**	**150**

Find the total cost incurred by the organisation for the three villages separately, using matrices.

Write one value generated by the organisation in the society.

Sol.

The number of attempt made in 3 villages X, Y, Z is written in Row matrix

$$\begin{vmatrix}400 & 300 & 100\\300 & 250 & 75\\500 & 400 & 150\end{vmatrix}$$

The cost for each mode per attempt (in column matrix)

$$\begin{vmatrix}50\\20\\40\end{vmatrix}$$

$$\text{Total cost}=\begin{vmatrix}400 & 300 & 100\\300 & 250 & 75\\500 & 400 & 150\end{vmatrix}\begin{vmatrix}50\\20\\40\end{vmatrix}$$

$$=\begin{vmatrix}20000+6000+4000\\15000+5000+3000\\25000+8000+6000\end{vmatrix}$$

$$=\begin{bmatrix}30000\\23000\\39000\end{bmatrix}$$

Total cost incurred by the organization for village $\mathrm{X}=\mathrm{Rs.}\,30{,}000$

Total cost incurred by the organization for village Y $=$ Rs. 23,000

Total cost incurred by the organization for village Z $=$ Rs. 39,000

Value generated by the organisation :

1. Prevention is better than cure so cleanliness leads to better quality of life (healthy life) It will help to avoid various diseases.
2. Concern for safety of women.

Q. 8. Solve for x : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Sol.

$$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$$

$$\tan^{-1}\left(\frac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right)=\tan^{-1}\frac{8}{31}$$

$$\left[\because \tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)\right.$$

$$\frac{2x}{1-(x^2-1)}=\frac{8}{31}$$

$$62x=8(2-x^2)$$

$$62x=16-8x^2$$

$$8x^2+62x-16=0$$

$$4x^2+31x-8=0 \qquad \textbf{(Dividing by 2)}$$

$$4x^2+32x-x-8=0$$

$$4x(x+8)-1(x+8)=0$$

$$(x+8)(4x-1)=0$$

$x+8=0$ or $4x-1=0$

$x=-8$ or $x=\frac{1}{4}$

Since $x=-8$ does not satisfy the given equation

$\therefore x=\frac{1}{4}$

Or

Prove the following :

$$\cot^{-1}\left(\frac{xy+1}{x-y}\right)+\cot^{-1}\left(\frac{yz+1}{y-z}\right)+\cot^{-1}\left(\frac{zx+1}{z-x}\right)=0 \qquad (0<xy,\ yx,\ zx<1)$$

Sol.

L. H. S. =

$$=\cot^{-1}\left(\frac{xy+1}{x-y}\right)+\cot^{-1}\left(\frac{yz+1}{y-z}\right)+\cot^{-1}\left(\frac{zx+1}{z-x}\right)$$

$$=\tan^{-1}\left(\frac{x-y}{1+xy}\right)+\tan^{-1}\left(\frac{y-z}{1+yz}\right)+\tan^{-1}\left(\frac{z-x}{1+zx}\right)$$

$$\left[\because \cot^{-1}\left(\frac{a}{b}\right)=\tan^{-1}\left(\frac{b}{a}\right)\right.$$

$$\left[\because \tan^{-1}\left(\frac{A-B}{1+AB}\right)=\tan^{-1}A-\tan^{-1}B\right]$$

$$=\tan^{-1}x-\tan^{-1}y+\tan^{-1}y-\tan^{-1}z+\tan^{-1}z-\tan^{-1}x$$

$=0=$ **R. H. S.**

Q. 9. Using properties of determinants, prove the following :

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Sol.

$$\text{L.H.S.} = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking common a, b, c from C_1, C_2 and C_3 respectively

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2 - C_3$

$$= abc\begin{vmatrix} -2c & c & a+c \\ 0 & b & a \\ -2c & b+c & c \end{vmatrix}$$

Taking common $(-2c)$ from C_1, we have

$$= -2abc^2\begin{vmatrix} 1 & c & a+c \\ 0 & b & a \\ 1 & b+c & c \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$

$$= -2abc^2\begin{vmatrix} 1 & c & a+c \\ 0 & b & a \\ 0 & b & -a \end{vmatrix}$$

Expanding along C_1, we have

$$= -2abc^2.1(-ab-ab)$$

$$= -2abc^2(-2ab)$$

$$= 4a^2b^2c^2 = \textbf{R.H.S.}$$

Q. 10. Find the ad joint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence show that $A.(adj\ A) = |A|I_3$.

Sol.

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$A_{11} = 1 + 4 = -3,$ $A_{12} = -(2+4) = -6$

$A_{13} = -4 - 2 = -6,$ $A_{21} = -(-2-4) = 6$

$A_{22} = 1 + 4 = 3,$ $A_{23} = -(2+4) = -6$

$A_{31} = 4 + 2 = 6,$ $A_{32} = -(2+4) = -6$

$A_{33} = -1 + 4 = 3$

$$adj\ A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$\mathbf{L.H.S.} = \text{A}.(adj\ \text{A})$

$$= \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$= 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (i)$$

Expanding along R_1 of matrix A,

$$|\text{A}| = -1(1-4) + 2(2+4) - 2(-4-2)$$

$$= 3 + 12 + 12 = 27$$

$\mathbf{R.H.S.} = |\text{A}|\text{I}_3$

$$= 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots (ii)$$

From ... (i) and ... (ii) $\mathbf{L.H.S.} = \mathbf{R.H.S.}$

Q. 11. Show that the function $f(x) = |x-1| + |x+1|$ for all $x \notin \text{R}$, is not differentiable at the points $x = -1$ and $x = 1$.

Sol.

$$|x-1| = \begin{cases} -(x-1), & x < 1 \\ +(x-1), & x \geq 1 \end{cases}$$

$$|x-1| = \begin{cases} -(x-1), & x < -1 \\ +(x-1), & x \geq -1 \end{cases}$$

$$f(x) = \begin{cases} -(x-1) - (x+1) & \text{for} \quad x < -1 \\ -(x-1) + (x+1) & \text{for} \quad -1 \leq x < 1 \\ (x-1) + (x+1) & \text{for} \quad x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} -2x, & \text{for} \quad x < -1 \\ 2, & \text{for} \quad -1 \leq x < 1 \\ 2x & \text{for} \quad x \geq 1 \end{cases}$$

At $x = -1$

(LHD at $x = -1$)

$= \lim_{x \to -1^{-1}} \frac{f(x) - f(-1)}{x - (-1)}$

$= \lim_{x \to -1^-} \frac{-2x-2}{(x-1)} [\because f(-1) = 2$

$= \lim_{x \to -1^-} \frac{2(x-1)}{(x-1)}$

$= \quad -2$

(RHD at $x = -1$)

$= \lim_{x \to -1^+} \frac{f(x) - f(-1)}{x - (-1)}$

$= \lim_{x \to -1^+} \frac{2-2}{x-1}$

$= \lim_{x \to -1^+} 0$

$= 0$

(LHD at $x = -1$) $\neq$ (RHD at $x = -1$)

So, $f(x)$ is not differentiable at $x = -1$

At $x = -1$

(LHD at $x = 1$)	(RHD at $x = 1$)
$= \lim_{x\to 1^{-1}} \frac{f(x)-f(1)}{x-1}$	$= \lim_{x\to 1^{+}} \frac{f(x)-f(1)}{x-1}$
$= \lim_{x\to 1^{-}} \frac{2-2}{x-1} \left[\begin{array}{l} \text{at } x = 1 \\ f(x) = 2x \\ f(1) = 2 \end{array}\right.$	$= \lim_{x\to 1^{+}} \frac{2x-2}{x-1}$
$= \lim_{x\to 1^{+}} 0$	$= \lim_{x\to 1^{-}} \frac{2(x-1)}{(x-1)}$
$= \quad 0$	$= \quad 2$

(LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$

Q. 12. If $y = e^{m\sin^{-1}x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$.

Sol.

$$y = e^{m\sin^{-1}x} \qquad \dots(i)$$

Differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = e^{m\sin^{-1}x}.\frac{m}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = y.\frac{m}{\sqrt{1-x^2}} \qquad \dots(ii)$$

$$\sqrt{1-x^2}\frac{dy}{dx} = my$$

Again differentiating both sides w.r.t. x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{1}{2\sqrt{1-x^2}}.(-2x) = m\frac{dy}{dx}$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{2\sqrt{1-x^2}}\frac{dy}{dx} = m\frac{dy}{dx}$$

$$\frac{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}}{\sqrt{1-x^2}} = m\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\left(\frac{ym}{\sqrt{1-x^2}}\right) \qquad \text{[From (ii)]}$$

$$\boldsymbol{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y = 0}$$

Q. 13. If $f(x) = \sqrt{x^2+1}$; $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x-3$, then find $f'[h'\{g'(x)\}]$.

Sol.

$$g(x) = \frac{x+1}{x^2+1}$$

Differentiating both sides w.r.t. x, we have

$$g(x) = \frac{(x^2+1).\frac{d}{dx}(x+1) - (x+1).\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1).1-(x+1).2x}{(x^2+1)^2}$$
$$= \frac{x^2+1-2x^2+2x}{(x^2+1)^2}$$
$$g(x) = \frac{-x^2-2x+1}{(x^2+1)^2}$$
$$h(x) = 2x - 3$$

Differentiating both sides w.r.t. x, we have

$$h'(x) = 2$$

$$\therefore h'(g'(x)) = 2 \quad \ldots (i) \quad \left[\begin{array}{l} \because \textbf{Let } h'^{(x)} = 2 \\ \textbf{then } h'^{(4)} = 2 \\ \textbf{and } h'(-6x+5) = 2 \end{array}\right.$$

$$f(x) = \sqrt{x^2+1}$$

Differentiating both sides w.r.t. x, we have

$$f(x) = \frac{1}{\sqrt{x^2+1}} \times 2x = \frac{x}{\sqrt{x^2+1}}$$

When $x = 2, f(x) = \frac{1}{\sqrt{2^2+1}} = \frac{2}{\sqrt{5}}$

$$f'[h'(g'(x))] = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}} \qquad \textbf{[From (i)]}$$

Q. 14. Evaluate : $\int (3-2x).\sqrt{2+x-x^2}\ dx$

Sol.

$$\int (3-2x).\sqrt{2+x-x^2}\ dx$$
$$= \int [2+(1-2x)]\sqrt{2+x-x^2}\ dx$$
$$= 2\int \sqrt{2+x-x^2}\ dx + \int (1-2x)\sqrt{2+x-x^2}\ dx$$
$$\textbf{[Let } p = 2x - x^2, dp = (1-2x)dx$$
$$= 2\int \sqrt{-(x^2-x-2)}\ dx + \int \sqrt{p\ dp}$$
$$= 2\int \sqrt{-\left[\left(x^2 - x + \left(\tfrac{1}{2}\right)^2 - \tfrac{1}{4} - 2\right]}\ dx + \int p^{\frac{1}{2}}\ dp$$
$$= 2\int \sqrt{-\left[\left(x-\tfrac{1}{2}\right)^2 - \tfrac{9}{4}\right]}\ dx + \int p^{\frac{1}{2}}\ dp$$
$$= 2\int \sqrt{\left(\tfrac{3}{2}\right)^2 - \left(x-\tfrac{1}{2}\right)^2}\ dx + \int p^{\frac{1}{2}}\ dp$$
$$= 2 \times \frac{1}{2}\left[\left(x-\frac{1}{2}\right)\sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2} + \frac{9}{4}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right)\right] + \frac{2}{3}p^{\frac{3}{2}} + C$$
$$\left[\because \int \sqrt{a^2+x^2}\ dx = \frac{1}{2}\left(x\sqrt{a^2+x^2} + a^2 \sin^{-1}\frac{x}{a}\right) + C\right.$$
$$= \left(\frac{2x-1}{2}\right)\sqrt{2+x+x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right) + \frac{2}{3}(2+x+x^2)^{\frac{3}{2}} + C$$

Or

Evaluate : $\int \frac{x^2+x+1}{(x^2+1)(x+2)} dx$

Sol.

Let $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$ ... (i)

$\Rightarrow x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

$\Rightarrow x^2+x+1 = A(x^2+1) + Bx(x+2) + C(x+2)$

Comparing the coefficients of x^2, x and constant terms,

$1 = A + B$ $\quad 1 = 2B + C$ $\quad 1 = A + 2C$

$1 - B = A$ ... (ii) $\quad 1 - 2B = C$ ... (iii) $\quad 1 = 1 - B + 2(1-2B)$ [From (ii) and (iii)]

$1 - \frac{2}{5} = A$ [From iv] $\quad 1 - 2\left(\frac{2}{5}\right) = C$ [From iv] $\quad 1 = 1 - B + 2 - 4B$

$1 - 1 - 2 = -5B - 2 = -5B$

$\therefore A = \frac{3}{5}$ $\quad C = \frac{1}{5}$ $\quad B = \frac{2}{5}$ ... (iv)

Putting the value of A, B and C in (i), we get

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{\frac{3}{5}}{x+2} + \frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1}$$

$$\therefore \int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx = \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x}{x^2+1} dx + \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{dp}{p} + \frac{1}{5}\int \frac{dx}{x^2+1} \quad \ldots \left[\begin{array}{l}\text{Let } p = x^2+1 \\ \therefore dp = 2x\,dx\end{array}\right.$$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|p| + \frac{1}{5}\tan^{-1} x + c$$

$$= \mathbf{\frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1} x + c}$$

$$\left(\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c\right)$$

Q. 15. Find : $\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

Sol.

$$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$$

$$= \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{\frac{2.2\tan x}{1+\tan^2 x}}} \quad \because \sin 2x = \frac{2\tan x}{1+\tan^2 x}$$

$$= \int_0^{\pi/4} \frac{dx}{\cos^3 x.2\sqrt{\frac{\tan x}{\sec^2 x}}} \quad \because 1+\tan^2 x = \sec^2 x$$

$$= \int_0^{\pi/4} \frac{dx}{2\cos^3 x\sqrt{\tan x}.\cos x}$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{\sec^2 x.\sec^2 x.}{\sqrt{\tan x}} dx = \frac{1}{2}\int_0^{\pi/4} \frac{(1+\tan^2 x)\sec^2 x\,dx}{\sqrt{\tan x}}$$

$$= \frac{1}{2}\int_0^1 \frac{(1+p^2)}{\sqrt{p}}\,dp \qquad \left[\begin{array}{l} \textbf{Let } p = \tan x \\ \quad dp = \sec^2 x\,dx \\ \textbf{When } x = \pi/4,\ p = 1 \\ \textbf{When } x = 0, p = 0 \end{array}\right.$$

$$= \frac{1}{2}\int_0^1 \left(\frac{1}{\sqrt{p}} + \frac{p^2}{\sqrt{p}}\right) dp$$

$$= \frac{1}{2}\int_0^1 \left(p^{-\frac{1}{2}} + p^{\frac{3}{2}}\right) dp = \frac{1}{2}\left[\frac{2}{1}p^{1/2} + \frac{2}{5}p^{5/2}\right]_0^1$$

$$= \frac{1}{2}\left[2(1)^{1/2} + \frac{2}{5}(1)^{5/2}\right] - \frac{1}{2}\left[2(0) + \frac{2}{5}(0)\right]$$

$$= \frac{1}{2}\left[2 + \frac{2}{5}\right] = \frac{1}{2}\left[\frac{10+2}{5}\right]$$

$$= \frac{1}{2}\left[\frac{12}{5}\right] = \frac{\mathbf{6}}{\mathbf{5}}$$

Q. 16. Find : $\int \frac{\log x}{(x+1)^2}\,dx$

Sol.

$$\int \frac{\log x}{(x+1)^2}\,dx$$

$$= \int \underset{\text{I}}{\log x}.\ \underset{\text{II}}{(x+1)^{-2}}\,dx$$

Integrating by parts taking $\log x$ as I[st] function, we have

$$= \log x. \int (x+1)^{-2}\,dx - \int \left(\frac{d}{dx}(\log x) \int (x+1)^{-2}\,dx\right) dx$$

$$= \log x. \frac{(x+1)^{-1}}{-1} - \int \frac{1}{x}.\frac{(x+1)^{-1}}{-1}\,dx$$

$$= \frac{-\log x}{x+1} + \int \frac{1}{x(x+1)}\,dx$$

$$= \frac{-\log x}{x+1} + \int \frac{dx}{x^2+x}$$

$$= \frac{-\log x}{x+1} + \int \frac{dx}{x^2+x+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$

$$= \frac{-\log x}{x+1} + \frac{1}{2.\frac{1}{2}}\log\left|\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}}\right| + \text{C}$$

$$\because \int \frac{dx}{x^2+a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$= \frac{-\log x}{x+1} + \log\left|\frac{x}{x+1}\right| + \text{C}$$

Q. 17. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath}$ **and** $\vec{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, **then find a unit vector perpendicular to both of the vectors** $(\vec{a} - \vec{b})$ **and** $(\vec{c} - \vec{b})$.

Sol.

$$\vec{a} - \vec{b} = (\hat{\imath} + 2\hat{\jmath} + 1\hat{k}) - (2\hat{\imath} + \hat{\jmath})$$
$$= -\hat{\imath} + \hat{\jmath} + \hat{k}$$
$$\vec{c} - \vec{b} = (3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}) - (2\hat{\imath} + \hat{\jmath})$$
$$= -\hat{\imath} - 5\hat{\jmath} - 5\hat{k}$$

Let $\vec{\text{D}} = (\vec{a} - \vec{b}) \times (\vec{c} - \vec{b})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix}$$

$= \hat{i}(-5+5) - \hat{j}(5-1) + \hat{k}\,(5-1)$

$= -4\hat{j} + 4\hat{k}$

$|\vec{D}| = \sqrt{16+16} = \sqrt{2 \times 16} = 4\sqrt{2}$

Required vector $= \pm \frac{\vec{D}}{|\vec{D}|}$

$= \pm \frac{(-4\hat{j}+4\hat{k})}{4\sqrt{2}}$

$= \pm \left(\frac{-4\hat{j}}{4\sqrt{2}} + \frac{4\hat{k}}{4\sqrt{2}}\right)$

$= \pm \left(\frac{-1}{\sqrt{2}}\hat{\boldsymbol{j}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{k}}\right)$

Q. 18. Find the equation of a line passing though the point $(1, 2, -4)$ and perpendicular to two lines $\vec{r} = (8\hat{\imath} - 19\hat{\jmath} + 10\hat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ and $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$

Sol.

The required line is perpendicular to the lines which are parallel to the vectors $\overrightarrow{b_1} = 3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}$ and $\overrightarrow{b_2} = 3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ respectively

So, it is parallel to the vector $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$

$$\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 5 & -5 \end{vmatrix}$$

$\vec{b} = \hat{\imath}(80-56) - \hat{\jmath}(-15-21) + \hat{k}(24+48)$

$= 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$

Thus, the required line passes thought the $(1,2,-4)$ and is parallel to the vector $\vec{b} = 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$ and $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

Vector equation of this line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$

or $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ ... where $\mu = 12\,\lambda$

Or

Find the equation of a line passing though the points $(-1, 2, 0), (2, 2, -1)$ and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

Sol.

Let a, b, c be direction of ratios of the normal to the plane.

Equations of plane through point $(-1,2,0)$ is

$a(x+1) + b(y-2) + c(z-0) = 0$ ... (i)

Point $(2,2,-1)$ lies on (i)

$a(2+1) + b(2-2) + c(-1-0) = 0$

$3a + 0.b - c = 0$ ... (ii)

Given line : $\frac{x+1}{1} = \frac{2\left(y+\frac{1}{2}\right)}{2} = \frac{z+1}{-1}$

$$\frac{x-1}{1} = \frac{\left(y+\frac{1}{2}\right)}{1} = \frac{z+1}{-1}$$

Direction ratios of given line are $1,1,-1$ Required plane is parallel to the given line

$a+b+c=0$ ... (iii)

$3a+0.b-c=0$

$a+b-c=0$

$\frac{a}{0+1} = \frac{-b}{-3+1} = \frac{c}{3-0}$

$\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = k(\text{let})$

$a=k, b=2k, c=3k$

Putting the value of a, b, c in (i), we have

$1(x+1)+2(y-2)+3z=0$

$x+1+2y-4+3z=0$

$\therefore$ $\boldsymbol{x+2y+3z-3=0}$ is the equation of the required plane.

Q. 19. 'Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution.

Sol.

The Number of spades is a random variable

Let it be denoted by X.

X can take values 0,1,2 or 3

Let $p = \text{P(a spade card)} = \frac{13}{52} = \frac{1}{4}$

$\therefore q = 1 - \frac{1}{4} = \frac{3}{4}$, Using $\text{P(r)} = n_{C_r}.q^{n-r}.p^r$

$\text{P}(\text{X}=0) = 3_{C_0}.q^{3-0}.p^0 = q^3 = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

$\text{P}(\text{X}=1) = 3_{C_1}.q^{3-1}p^1 = 3q^2p = 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{27}{64}$

$\text{P}(\text{X}=2) = 3_{C_2}.q^{3-2}.p^2 = 32qp^2 = 3\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 = \frac{9}{64}$

$\text{P}(\text{X}=3) = 3_{C_3}.q^{3-3}.p^3 = p^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

$\mathbf{X}_i$	0	1	2	3
$\mathbf{P}_i$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$\mathbf{X}_i$	$\mathbf{P}_i$	$\mathbf{P}_i\mathbf{X}_i$
0	$\frac{27}{64}$	0
1	$\frac{27}{64}$	$\frac{27}{64}$

2	$\frac{9}{64}$	$\frac{18}{64}$
3	$\frac{1}{64}$	$\frac{3}{64}$
		$\sum P_i X_i = \frac{48}{64}$

Mean$= \sum P_i X_i = \frac{48^3}{64_4} = \frac{3}{4}$

Or

For 6 trials of an experiment, let X be a binomial variants which satisfies the relation $9P(X=4) = P(X=2)$Find the probability of success.

Sol.

Let p be the probability of success.

$n = 6 \ P(r) = n_{C_r}.q^{n-r}p^r$

$9P(X=4) = P(X=2)$

$9\left[6_{C_4}.q^{6-4}p^4\right] = 6_{C_4}.q^{6-2}p^2$

$9.6_{C_4}.q^2p^4 = 6_{C_4}.q^4p^2$ $\left[\because 6_{C_4} = 6_{C_2}\right]$

$9p^2 = q^2$

Taking square-root, we get

$q = 3p$

As $0 \le p \le 1$

$0 \le p \le 1$

Now $p + q = 1$

$p + 3q = 1$

$4q = 1$

$q = \frac{1}{4}$

$\therefore$ **The probability of success, $q = \frac{1}{4}$**

Section – C

Question numbers 20 to 26 carry 6 marks each.

Q. 20. Consider $f: R_+ \to [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54+5y}-3}{5}\right)$.

Sol.

$f(x) = 5x^2 + 6x - 9$ (given)

Let $f(x) = y \Rightarrow y = 5x^2 + 6x - 9$

$5y = 25x^2 + 30x - 45$ **(Multiplying both sides by 5)**

$5y = 25x^2 + 30x + (3)^2 - 45 - (3)^2$

$5y = (5x+3)^2 - 54$ ... (i)

$5y + 54 = (5x+3)^2$

$\sqrt{5y+54} = 5x + 3$ **(Square root on both sides)**

$\sqrt{5y+54} - 3 = 5x$

$x = \frac{\sqrt{5y+54}-3}{5}$

$g(y) = \frac{\sqrt{5y+54}-3}{5}$ ... (ii)

$\Rightarrow gof(x) = g(f(x))$

$= g(5x^2 + 6x - 9)$

$= \frac{\sqrt{5(5x^2+6x-9)+54}-3}{5}$

$= \frac{\sqrt{25x^2+30x-45+54}-3}{5}$

$= \frac{\sqrt{25x^2+30x+9}-3}{5}$

$gof(x) = \frac{\sqrt{(5x+3)^2}-3}{5} = \frac{5x+3-3}{5} = x$ ... (iii)

and $fog(y) = f(g(y))$

$= f\left(\frac{\sqrt{5y+54}-3}{5}\right)$ **[From (ii)**

$\frac{\left[5\left(\frac{\sqrt{5y+54}-3}{5}\right)+3\right]^2-54}{5}$

From (i)

$5y = (5x+3)^2 - 54$

$y = \frac{(5x+3)^2-54}{5}$

$f(x) = \frac{(5x+3)^2-54}{5}$

$= \frac{\left(\sqrt{5y+54}\right)^2-54}{5} = \frac{5y+54-54}{5} = \frac{5y}{5} = y$

$fog(y) = y$ ... (iv)

From (iii) and (iv), f is invertible and $f^{-1} = g$

$f^{-1}(y) = \frac{\sqrt{54+5y}-3}{5}$ **[From (ii)**

Or

A binary operation * is defined on the set $x = \mathbf{R} - \{-1\}$ by

$x * y = x + y + xy, \forall\, x, y \notin \mathbf{X}$.

Check whether » is commutative and associative. Find the identity element and also find the inverse of each element of X.

Sol.

(i) $x * y = x + y + xy, , \forall\, x, y \notin \mathrm{X}$

$y * x = y + x + yx$

$= x + y + xy = x * y$

Hence '*'is associative

(ii)$x * (y * z) = x * (y + z + yz)$

$= x + (y + z + yz) + x(y + z + yz)$

$= x + y + z + yz + xy + zx + xyz$ ... (i)

$(x * y) * z = (x + y + xy) * z$

$= x + y + xy + z + (x + y + xy)z$

$= x + y + xy + z + xz + yz + xyz$ ... (ii)

From (i) and $(ii)$$x * (y * z) = (x * y) * z$

Hence '*' is associative

Let $'e'$ be the identity element of X

Then $x * e = x + e + xe = x \; \forall \; x \in \mathrm{X}$

$x + e + xe = x$

$\Rightarrow e(1 + x) = 0$

$\Rightarrow e = 0$ or $1 + x = 0$

$e = 0$ or $x = -1$, Which is not possible because $x \in \mathrm{R} - \{-1\}$ (given)

$\therefore e = 0$

Clearly $\mathrm{X} * 0 = x + 0 + 0 = x = 0 + x, \forall \; x \in \mathrm{X}$

$\therefore$ '0' is the identity element of X ... (i)

Let $y \in x$ be the inverse of $x \in \mathrm{X}$

Then $x * y = y * x = e$

$x + y + xy = 0$

$x + y(1 + x) = 0$

$y(1 + x) = -x$

$\Rightarrow y = \frac{-x}{1+x}$ or $x^{-1} = \frac{-x}{1+x}$

Q. 21. Find the value of p for when the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

Sol.

$x^2 = 9p(9 - y)..(i)$ $\quad$ $x^2 = p(y + 1) ... (ii)$

From (i) and (ii),

$9\not{p}(9 - y) = \not{p}(y + 1)$

$\Rightarrow 81 - 9y = y + 1$

$\Rightarrow 81 - 1 = y + 9y$

$\Rightarrow 80 - 10y \Rightarrow y = 8$

Putting the value of y in (i), we have

$x^2 = 9p(9 - 8)$

$x^2 = 9p$ $\quad$ $\Rightarrow x = \pm 3\sqrt{p}$

$\therefore$ Point of intersection $\mathrm{A}\left(\pm 3\sqrt{\mathrm{p}}, 8\right)$

$x^2 = 9p(9 - y)$

Differentiating w.r.t. x, we have

$2x = 9p\left(-\frac{dy}{dx}\right)$

$m_1 = \frac{dy}{dx} = \frac{-2x}{9p}$

$x^2 = p(y + 1)$

Differentiating w.r.t. x, we have

$2x = p\frac{dy}{dx}$

$m_2 = \frac{dy}{dx} = \frac{2x}{p}$

If the curve cut at right-angles,

then $m_1 \times m_2 = -1$

$= \frac{-2x}{9p} \times \frac{2x}{p} = -1$

$\Rightarrow -9p^2 = -4x^2$

$\Rightarrow 9p^2 = 4(9p)$ $\quad$ $\left(\mathrm{Put}\; x = \pm\sqrt{\mathrm{p}}\right)$

$\Rightarrow 9p^2 - 36p = 0$

$\Rightarrow 9p(p-4) = 0$

$\Rightarrow 9p = 0$ or $p - 4 = 0$

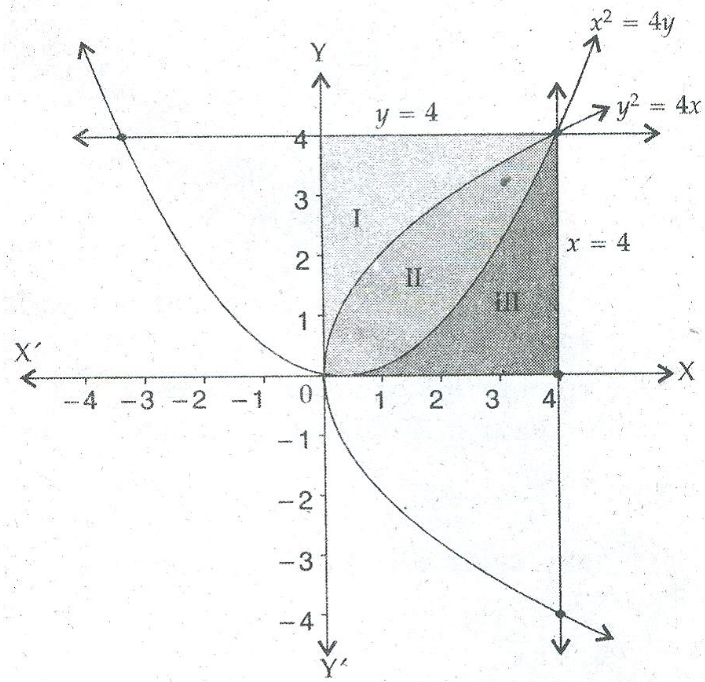
$\Rightarrow \boldsymbol{p = 0}$ **or** $\boldsymbol{p = 4}$

Q. 22. Using integration, prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

Sol.

$y^2 = 4x \quad \Rightarrow y = 2\sqrt{x} \quad \dots (i)$

$x^2 = 4y \quad \Rightarrow y = \frac{x^2}{4} \quad \dots (ii)$



For point of intersection

$x^2 = 4y$

$\left(\frac{y^2}{4}\right)^2 = 4y \qquad$ [from$(i) y^2 = 4x$ or $\frac{y^2}{4} = x$

$y^4 = 64y$

$y^4 - 64y = 0$

$\Rightarrow y(y^3 - 64) = 0$

$\therefore y = 0$ or $y^3 - 64 \Rightarrow y = 4$

Shaded area (I) $= \int_0^4 x\, dy$

$= \int_0^4 \frac{y^2}{4} dy$

$= \frac{1}{4} \cdot \frac{1}{3} [y^3]_0^4$

$= \frac{1}{12}(4^3 - 0^3)$

$= \frac{64}{12} = \boldsymbol{\frac{16}{3}}$ **sq. units** $\quad \dots (i)$

Shaded area (II) $= \int_0^4 2\sqrt{x}\,dx - \int_0^4 \frac{x^2}{4}\,dx$

$= 2.\frac{2}{3}\left[x^{3/2}\right]_0^4 - \frac{1}{4}.\frac{1}{3}[x^3]_0^4$

$= \frac{4}{3}\left[4^{3/2} - 0\right]_0^4 - \frac{1}{12}[4^3 - 0]$

$= \frac{4}{3} \times 2 \times \frac{3}{2} - \frac{1}{12}[64]$

$= \frac{32}{3} - \frac{16}{3} = \mathbf{\frac{16}{3}}$ **sq. units** ... (ii)

Shaded area (III) $= \int_0^4 \frac{x^2}{4} dx$

$= \frac{1}{4}.\frac{1}{3}[x^3]_0^4$

$= \frac{1}{12}[4^3 - 0]$

$= \frac{64}{12} = \mathbf{\frac{16}{3}}$ **sq. units** ... (iii)

From (i),(ii) and (iii),

Area bounded by curves $y^2 = 4x$ and $x^2 = 4y$ divides the area of square in three equal parts.

Q. 23. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

Sol.

$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ ... (i)

Dividing numerator and denominator by x^2,

$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x} - 1} = g\left(\frac{y}{x}\right)$ ... (ii)

R.H.S. of differential equation (ii) is of the from $g\left(\frac{y}{x}\right)$ and so it is a homogeneous.

Therefore, Equation (i) is a homogeneous differential equation

$\left[\mathbf{Let}\ y = vx \Rightarrow \frac{y}{x} = v\right.$

$\left.\frac{dy}{dx} = v.1 + x.\frac{dv}{dx}\right]$

$v + x\frac{dy}{dx} = \frac{v^2}{v-1}$

$x\frac{dy}{dx} = \frac{v^2}{v-1} - v$

$x\frac{dy}{dx} = \frac{v^2 - v^2 + v}{v-1}$

$\frac{v-1}{v} dv = \frac{dy}{x}$ ... (iii)

Integrating both sides of equation (iii), we get

$\int\left(\frac{v^1}{v} - \frac{1}{v}\right) dv = \int \frac{dx}{x}$

$v - \log|v| = \log|x| + C$

$\frac{y}{x} - \log\left|\frac{y}{x}\right| - \log|x| = C$

$\frac{y}{x} - \left[\log\left|\frac{y}{x}.x\right|\right] = C$ $[(\because \log m + \log n = \log mn)$

$\frac{y}{x} - \log|y| = C$

$\frac{y-\log|y|}{x} = C$

$y - x\log|y| = Cx$ which is the general solution of the given differential solution.

Or

Find the particular solution of the differential equation $(\tan^{-1} y - x)dy = (1 + y^2)\,dx$, given that $x = 1$ when $y = 0$.

Sol.

$(\tan^{-1} y - x)dy = (1 + y^2)dx$

$\frac{\tan^{-1} y - x}{1+y^2} = \frac{dx}{dy}$

$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2} - \frac{x}{1+y^2}$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$

Comparing with $\frac{dy}{dx} + Px = Q$

$\therefore\ P = \frac{1}{1+y^2},\quad Q = \frac{\tan^{-1} y}{1+y^2}$

$I.F. = e^{\int P\,dy} = e^{\int \frac{\tan^{-1} y}{1+y^2}} = e^{\int \tan^{-1} y}$

Hence the solution is

$x(I.F.) = \int Q.(I.F.)dy$

$x.e^{\int P\,dy} = \int \frac{\tan^{-1} y}{1+y^2}.e^{\int \tan^{-1} y}dy$

$= \int \underset{I\ \ II}{t.e^t dt}$ $\quad\left[\begin{array}{l}\textbf{Let } t = \tan^{-1} y \\ dt = \frac{1}{1+y^2}dy\end{array}\right.$

$x.e^{\int \tan^{-1} y} = t.e^t - \int 1.e^t dt$

$= t.e^t - e^t + C$

$= e^t(t-1) + C$

$x.e^{\int \tan^{-1} y} = e^{\int \tan^{-1} y}(\tan^{-1} y - 1) + C$

$x = \tan^{-1} y - 1 + \frac{C}{e^{\tan^{-1} y}}$ $\quad \ldots(i)\quad [\because x = 1, y = 0]$

$1 = \tan^{-1}(0) - 1 + \frac{C}{e^{\tan^{-1} y}}$

$2 = 0 + \frac{C}{1}$ $\quad [\because e^0 = 1]$

$C = 2$

Putting the value of C in (i), we have

$x = \tan^{-1} y - 1 + \frac{2}{e^{\tan^{-1} y}}$

$\boldsymbol{x = (\tan^{-1} y - 1) + 2.e^{\tan^{-1} y}}$

which is the particular solution of given differential equation.

Q. 24. Find the distance of the point $P(3,4,4)$ from the point, where the line joining the points $A(3,-4,-5)$ and $B(2,-3,1)$ intersects the plane $2x+y+z=7$.

Sol.

Direction ratios of line AB are

$$x_2 - x_1, \quad y_2 - y_1, \quad z_2 - z_1$$
$$2-3, \quad -3+4, \quad 1+5$$
$$-1, \quad 1 \quad 6$$

Equation of line AB is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$
$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = m(\text{let}) \qquad \textbf{(Using Pt. A)}$$

Let point $C(-m+3, m-4, 6m-5)$ ... (i)

be the point of intersection between line AB and given plane

So point C lies on the given plane

$$2x+y+z=7$$
$$2(-m+3)+(m-4)+(6m-5) \qquad ...(i)$$
$$-2m+6+m-4+6m-5=7$$
$$5m-3=7 \Rightarrow 5m+10 \Rightarrow m=\frac{10}{5}=2$$

Putting the value of m in (i),

Coordinates of point $C = (-2+3, 2-4, 12-5)$ or $(1,-2,7)$

Point P is $(3,4,4,)$

Required distance $=$ CP

$$= \sqrt{(3-1)^2+(4+2)^2+(4-7)^2}$$
$$= \sqrt{4+36+9} = \sqrt{49} = \textbf{7 units}$$

Q. 25. A company manufacturers three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs. 12,000 and of factory II is Rs. 5,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

Sol.

	No of days	*Calcutators*			*Cost*
		A	*B*	*C*	
Fecories I	x	50	50	30	Rs. 12,000
Fecories II	y	40	20	40	Rs. 15,000
		6400	6400	4800	
		At least	At least	At least	

$$Z_{\min} = (12000x + 15000y)$$

Subjects to the constraints :

$$50x + 40y \geq 6400$$
$$50x + 20y \geq 4000$$

$30x + 40y \geq 4800$ $x, y \geq 0$

$50x + 40y \geq 6400$

Let $50x + 40y \geq 6400$

or $5x + 4y = 640$

... (i)

x	0	128
y	160	0

$50x + 20y \geq 4000$

Let $50x + 20y \geq 4000$

or $5x + 2y = 400$

... (ii)

x	0	80
y	200	0

$30x + 40y \geq 4800$

Let $30x + 40y \geq 4800$

or $3x + 4y = 480$

... (iii)

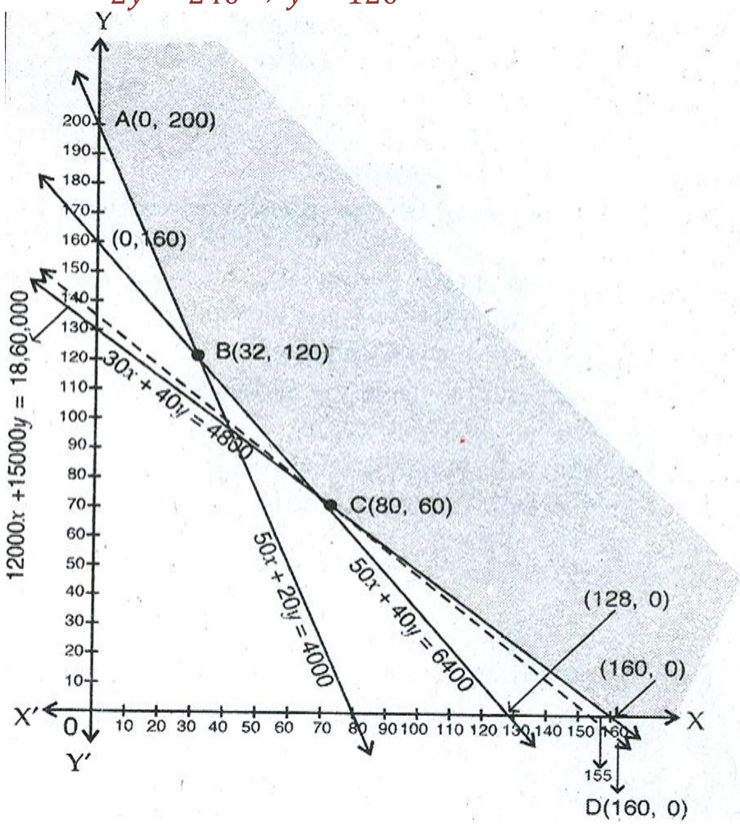
x	0	160
y	120	0

Om solving (i) and (ii)

$5x + 4y = 640$ ... (i)

$-\ 5x \pm 2y = 400$ ... (ii)

$2y = 240 \Rightarrow y = 120$



Putting the value of y in (ii),

$5x + 240 = 400;$ $5x = 160$ $\Rightarrow x = 32$

$\therefore$ Point B(32,120)

Om solving (i) and (ii)

$5x + 4y = 640$ ... (i)

$-\ 3x \pm 4y = 480$ ... (iii)

$2x \quad = 160 \Rightarrow x = 80$

Putting the value of x in (i),

$5(80) + 4y = 640$ $4y = 640 - 400$

$y = \frac{240}{4} = 60$

$\therefore$ Point C(80,60)

Corner Points	$Z = 12{,}000x + 15{,}000y$
A(0,200)	$15000(200) = 30{,}00{,}000$
B(32,120)	$12000(32) + 15000(120) = 21{,}84{,}000$
C(80,60)	$12000(80) + 15000(60) = 10{,}60{,}000 \leftarrow$ Smallest
D(160,0)	$12000(160) = 19{,}20{,}000$

From, the table, we find 18,60,000 is the smallest value of Z the corner point C(80,60)

Here is the feasible region is unbounded.

Therefore, 18,60,000 may or may not be the minimum value of Z to decide this issue, graph the inequality

$12000x + 15000y < 1860000$

Let $12000x + 15000y = 1860000$

or $12x + 15y = 1860$

or $4x + 5y = 620$

It has not common points

So, Rs. 18,60,000 will be the C(80,60)

Factory $\mathbf{I}$ = **80 Days**

Factory $\mathbf{II}$ = **60 Days**

Minimum cost = $\mathbf{Rs. 18, 60, 000}$

Q. 26. In a factory which manufacturers bolts, machine A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

Sol.

Let E_1 : the Bolt is manufactured by machine A

Let E_2 : the Bolt is manufactured by machine B

Let E_3 : the Bolt is manufactured by machine C

$\therefore P(E_1) = \frac{30}{100}, P(E_2) = \frac{50}{100}, P(E_3) = \frac{20}{100}$

$= \frac{3}{10} \qquad = \frac{5}{10} \qquad = \frac{2}{10}$

Let the event A be the bolt is defective

$P(A/E_1) = \frac{3}{100}, P(A/E_2) = \frac{4}{100}, P(A/E_3) = \frac{1}{100}$

P (bolt is not manufactured by machine B)

= P (bolt is manufactured by machine A or C)

$= P(\overline{E_2}/A)$

$$= \frac{P(E_1)\times P\ (A/E_1)+P(E_3)\times\ P\ (A/E_3)}{[P(E_1)\times P\ (A/E_1)]+[P(E_2)\times P\ (A/E_2)]+[P(E_3)\times\ P\ (A/E_3)]}$$

$$= \frac{\left(\frac{3}{10}\times\frac{3}{100}\right)+\left(\frac{2}{10}\times\frac{1}{100}\right)}{\left(\frac{3}{10}\times\frac{3}{100}\right)+\left(\frac{5}{10}\times\frac{4}{100}\right)+\left(\frac{2}{10}\times\frac{1}{100}\right)}$$

$$= \frac{\frac{9+2}{1000}}{\frac{9+20+2}{1000}} = \mathbf{\frac{11}{31}}$$